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ON THE INFLUENCE OF HEAT TRANSFER AND BLOWING ON THE STRUCTURE OF LAMINAR HYPERSONIC FLOW BEHIND A BODY*

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The asymptotic solution of the Navier-Stokes equations in the problem of hypersonic flow at large distances behind a body of finite size is studied. The principal term of the expansion for the perturbation of the longitudinal velocity component in the region of the trace, decaying exponentially in the direction of the outer boundary of the trace, is derived. The stream function is assumed constant along the axis of the trace and, generally speaking, non-zero. The solution constructed, enables not only the effect of the resistance, but also of the heat and mass flux from the streamlined body to be taken into account. The second terms of the asymptotic expansions of the flow parameters are obtained for the whole region of perturbed motion.

1. Consider a plane parallel ($\nu = 1$) or axisymmetric ($\nu = 2$) hypersonic ($p_\infty = 0$) flow of an ideal gas past a body of finite size. The coefficient of viscosity λ_1, λ_2 and thermal conductivity k are assumed to be proportional to the specific enthalpy, and the ratio $\varkappa$ of the specific heats c_p and c_v is assumed to be constant and to satisfy the inequality $1 < \varkappa < 2$. We denote the Prandtl number by $N_{\mathrm{Pr}} = c_p\lambda_1/k$. We regard all quantities as dimensionless and use as the basic units ρ_∞, U_∞ and λ_{10} which denote respectively, the density of the oncoming flow, its velocity, and the ratio of the first coefficient of viscosity to the specific enthalpy.

We introduce velocity perturbations, denoting by $1 + v_x$, v_r the components of the velocity vector along the x, r -axes of a plane or cylindrical system of coordinates, the origin of which coincides with the body. We denote the pressure, density and specific enthalpy by p, ρ, w respectively and we take, as the basic system, the system of Navier-Stokes equations written in Mieses variables x, Ψ. The aim of this paper is to construct an asymptotic solution of the system as $x \to \infty$, taking into account the energy and mass flux from the streamlined body.

The principal terms of the asymptotic expansions neglecting such heat and mass transfer were obtained in /1/. The solution constructed there included two, essentially different regions of flow, the outer region and the laminar trace. In the first region the effect of viscosity and thermal conductivity can be neglected and the flow is determined basically by the wave resistance force F_0. We can assume within the approximation used /2/ that the outer region is separated from the oncoming flow by a shock wave, which as $x \to \infty$ takes the form

$$r_s = Cx^m + \ldots, \quad m = \frac{2}{2+\nu}, \quad C = (r_0F_0)^{1/(2+\nu)}$$

The subsequent expansions of the flow parameters depend on the selfsimilar variable

$$\eta = \frac{\nu}{C^\nu}\Psi x^{-\nu m}$$

and are given by the formulas

$$\begin{gathered} v_x = v_{x0}x^{2(m-1)}v_{x11}(\eta) + \ldots, \quad v_r = v_{r0}x^{m-1}v_{r11}(\eta) + \ldots \\ p = p_0x^{2(m-1)}p_{11}(\eta) + \ldots, \quad \rho = \rho_0\rho_{11}(\eta) + \ldots \\ w = w_0x^{2(m-1)}w_{11}(\eta) + \ldots, \quad r = Cx^m r_{11}(\eta) + \ldots \end{gathered} \tag{1.1}$$

in which

$$\begin{gathered} p_0 = -v_{x0} = \frac{8C^2}{(2+\nu)^2(\varkappa+1)}, \quad v_{r0} = \frac{4C}{(2+\nu)(\varkappa+1)} \\ \rho_0 = \frac{\varkappa+1}{\varkappa-1}, \quad w_0 = \frac{8\varkappa C^2}{(2+\nu)^2(\varkappa+1)^2} \\ r_0 = \nu(2+\nu)^2(\varkappa+1)^2\left\{16\pi^{\nu-1}\int_0^1[v_{r11}^2(\eta) + w_{11}(\eta)]\,d\eta\right\}^{-1} \end{gathered}$$

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Using the fact that such motion is analogous to the isoenergetic motion of a gas /3/, we can obtain the functions with subscripts 11 from the exact solutions of the problem of an intense explosion /4/ written in Lagrange variables.

In order to analyze the flow in the region of a viscous, heat-conducting trace, the following variable was used in /1/:

$$\zeta = \frac{\nu}{C^\nu} \Psi x^{-\nu m+\delta}, \quad \delta = \frac{\nu\varkappa}{2(\varkappa+\nu-1)}$$

By passing to the limit as $\eta \to 0$ in the expansions (1.1) written in inner variables x, ζ, we were able to determine the form of the principal terms of the inner expansions

$$\begin{gathered} v_x = v_{x0} x^{2(m-1)+\delta/\varkappa} v_{x21}(\zeta) + \ldots, \quad v_r = v_{r0} x^{m-1-\delta(\varkappa-1)/\nu\varkappa} v_{r21}(\zeta) + \ldots \\ p = p_0 x^{2(m-1)} p_{21}(\zeta) + \ldots, \quad \rho = \rho_0 x^{-\delta/\varkappa} \rho_{21}(\zeta) + \ldots \\ w = w_0 x^{2(m-1)+\delta/\varkappa} w_{21}(\zeta) + \ldots, \quad r = C x^{m-\delta(\varkappa-1)/\nu\varkappa} r_{21}(\zeta) + \ldots \end{gathered} \tag{1.2}$$

and also obtain limiting expressions for the functions with subscripts 21 as $\zeta \to \infty$. The latter were used to construct a numerical solution of the boundary value problem describing the flow within the trace. Note that expansions (1.2) can be written in a slightly different form from those in /1/. The present form clarifies the form of the dependence of the inner solution ion the wave resistance F_0.

2. Below we consider gas flows in which the streamlined body transmits to the stream certain finite amounts of energy and mass of the gas per unit time. To construct a new asymptotic solution we alter the form of the inner expansion

$$v_x = C_u x^n v_{x22}(\zeta) + \ldots, \quad C_u = \text{const} \tag{2.1}$$

and require that integration of this perturbation over Ψ across the trace should give a result independent of the distance x from the plane of integration. This condition immediately yields the value

$$n = -\frac{2\nu}{2+\nu} + \frac{\nu\varkappa}{2(\varkappa+\nu-1)}$$

so that the principal term in the expansion for v_x given in (2.1) is of higher order in x than that given in (1.2). The above changes in v_x do not affect the principal terms of the expansions of other flow parameters. They retain their previous form (1.2), since the equation for determining the function v_{x22}

$$\begin{gathered} \frac{K}{n} \frac{d}{d\zeta} \left[r_{21}^{2(\nu-1)} \frac{dv_{x22}}{d\zeta} \right] - \zeta \frac{dv_{x22}}{d\zeta} - v_{x22} = 0 \\ K = \frac{8\nu^2\varkappa p_{21}}{(2+\nu)^2(\varkappa^2-1)}, \quad p_{21} = \text{const} \end{gathered} \tag{2.2}$$

is separated from the remaining equations of the first-approximation system and its solution is obtained after the functions $v_{r21}, p_{21}, \rho_{21}, w_{21}, r_{21}$ have been determined.

The solution of (2.2) satisfying the symmetry condition has the form (the multiplicative integration constant is taken into the expansion (2.1))

$$v_{x22}(\zeta) = \exp\left[-\frac{n}{K} \int_0^\zeta r_{21}^{-2(\nu-1)}(\xi)\, \xi d\xi \right]$$

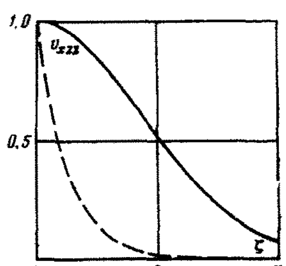

Fig.1

Fig.1 shows the dependence of v_{x22} on ζ for $\varkappa = 1.4$, $N_{\Pr} = 0.75$ and for $\nu = 1$ (the solid line) and $\nu = 2$ (the dashed line). We see at once that the solution does not violate the condition for matching with the outer expansions (1.1) since it is exponentially small in x, in the domain of finite η.

In order to describe the flows carrying additional gaseous mass, we must stipulate the symmetry conditions for $\Psi = \Psi_0$, generally speaking different from zero, and consider the subregion of the trace in which x, Ψ will play the part of characteristic variables. By passing to the limit as $\zeta \to 0$ in the inner expansions written in terms of the Mieses variables, we can obtain the expansions in the new subregion in the form

$$\begin{gathered} v_x = C_u x^n v_{x31}(\Psi) + \ldots, \quad v_r = v_{r0} x^{-1+\delta/\nu\varkappa} v_{r31}(\Psi) + \ldots \\ p = p_0 x^{2(m-1)} p_{31}(\Psi) + \ldots, \quad \rho = \rho_0 x^{-\delta/\varkappa} \rho_{31}(\Psi) + \ldots \\ w = w_0 x^{2(m-1)+\delta/\varkappa} w_{31}(\Psi) + \ldots, \quad r = C x^{\delta/\nu\varkappa} r_{31}(\Psi) + \ldots \end{gathered} \tag{2.3}$$

and also give the limiting conditions for the functions with subscripts 31 as $\Psi \to \infty$. Substituting the expansions (2.3) into the Navier-Stokes equations we obtain a first-approximation system, the solution of which satisfies the matching and symmetry conditions and has the form

$$v_{x31}(\Psi) = v_{x22}(0), \quad v_{r31}(\Psi) = \frac{(2+\nu)(\varkappa+1)}{8(\varkappa+\nu-1)} r_{31}(\Psi), \quad p_{31}(\Psi) = p_{21}$$

$$\rho_{31}(\Psi) = \rho_{21}(0), \quad w_{31}(\Psi) = w_{21}(0), \quad r_{31}(\Psi) = \left[\frac{\nu}{C^\nu}\frac{d\sigma_{21}}{d\zeta}(0)(\Psi - \Psi_0)\right]^{1/\nu}, \quad \sigma_{21} = r_{21}^\nu$$

Next we compute the total resistance force F_x, the energy E_t and mass of gas M_t released by the body per unit time. As a result we obtain

$$F_x = \frac{C^{2+\nu}}{r_0} - \frac{2\pi^{\nu-1}C^\nu C_u}{\nu}\int_0^\infty v_{x22}(\zeta)\,d\zeta + 2\pi^{\nu-1}\Psi_0 \tag{2.4}$$

$$E_t = \frac{2\pi^{\nu-1}C^\nu C_u}{\nu}\int_0^\infty v_{x22}(\zeta)\,d\zeta - \pi^{\nu-1}\Psi_0, \quad M_t = -2\pi^{\nu-1}\Psi_0$$

The value of the integral of v_{x22} at $\varkappa = 1.4$ and $N_{\mathrm{Pr}} = 0.75$ is 2.183 for $\nu = 1$ and 0.5340 for $\nu = 2$.

The solutions (1.1), (1.2), (2.1) and (2.3) depend on three constants, namely C, C_u and Ψ_0. Equations (2.4) enable us to express these constants in terms of the integral characteristics of the processes taking place on the body. Solving (2.4) for C, C_u and Ψ_0 we obtain

$$\Psi_0 = -\frac{1}{2\pi^{\nu-1}}M_t, \quad C = \left[r_0\left(F_x + E_t + \frac{1}{2}M_t\right)\right]^{1/(2+\nu)}$$

$$C_u = \left(E_t - \frac{1}{2}M_t\right)\left[r_0\left(F_x + E_t + \frac{1}{2}M_t\right)\right]^{-\nu/(2+\nu)} \times \left[\frac{2\pi^{\nu-1}}{\nu}\int_0^\infty v_{x22}(\zeta)\,d\zeta\right]^{-1}$$

We will now write the following expression for the wave resistance force

$$F_0 = F_x + E_t + {}^1/_2 M_t$$

In the absence of energy emission and blowing, the wave resistance at the body becomes identical to the total resistance force. On the other hand, even in the case when the total resistance force $F_x = 0$, the wave resistance may differ from zero and be defined by the heat flux E_t and the mass flux M_t emerging from the streamlined body. Thus the expansions (1.1) and (2.1), the last five expansions of (1.2) and expansions (2.3) together yield the first approximation to the asymptotic solution, taking into account heat transfer and blowing. Higher-order approximations must however be considered to explain the effect of these phenomena on the structure of the flow in the outer region and on the form of the expansions for v_r, p, ρ, w, r.

3. We will introduce into the inner expansions (1.2) correction terms, replacing the factors $q_{21}(\zeta)$ by the expressions $q_{21}(\zeta) + x^n q_{23}(\zeta)$, where $q = v_r, p, \rho, w, r$. Substitution of new expansions into the Navier-Stokes equations yields an inhomogeneous linear system of ordinary differential equations for the functions q_{23}, the free terms of which contain the function v_{x22}. Introducing the auxilliary function $\sigma_{23} = \nu r_{23} r_{21}^{\nu-1}$ and taking into account the condition

$$\sigma_{23}(0) = -\frac{\nu}{C^\nu}\frac{d\sigma_{21}}{d\zeta}(0)\,\Psi_0$$

which follows from the solution (2.3), we can reduce the system to a single inhomogeneous linear second-order equation

$$\frac{K}{nN_{\mathrm{Pr}}}\left[\sigma_{21}^{\nu-1}\frac{d^2\sigma_{23}}{d\zeta^2} + (\nu-1)\sigma_{23}\frac{d^2\sigma_{21}}{d\zeta^2}\right] - \zeta\frac{d\sigma_{23}}{d\zeta} - \frac{\sigma_{23}}{\varkappa} = -\frac{KC_u}{nN_{\mathrm{Pr}}}\left[\sigma_{21}^{\nu-1}\frac{d}{d\zeta}\left(v_{x22}\frac{d\sigma_{21}}{d\zeta}\right) + \sigma_{21}^{\nu-1}v_{x22}\frac{d^2\sigma_{21}}{d\zeta^2}\right] + C_u v_{x22}\zeta\frac{d\sigma_{21}}{d\zeta} - \Psi_0\frac{\nu-2}{\varkappa C}\frac{d\sigma_{21}}{d\zeta}(0) + \frac{C_u}{\varkappa}\int_0^\zeta v_{x22}(\xi)\frac{d\sigma_{21}}{d\zeta}(\xi)\,d\xi \tag{3.1}$$

To construct the solution of (3.1) we must formulate an additional condition at $\zeta \to \infty$, which can be done by analyzing the higher-order approximations in the outer region. However, the principal term of the expansion for σ_{23} can be obtained without this condition when $\zeta \to \infty$. The term is determined from the particular solution of the inhomogeneous equation (3.1). As a result we have

$$\sigma_{23}(\zeta)=C_w+\ldots,\quad C_w=\Psi_0\frac{\nu-2}{C}\frac{d\sigma_{21}}{d\zeta}(0)-C_u\int_0^\infty v_{x22}(\zeta)\frac{d\sigma_{21}}{d\zeta}(\zeta)\,d\zeta \tag{3.2}$$

Computing the coefficients in the expression for C_w at $\varkappa=1.4$, $N_{P_r}=0.75$, we have $C_w=-0.4947\Psi_0/C-0{,}9343\,C_u$ for $\nu=1$ and $C_w=-0.3037C_u$ for $\nu=2$. The asymptotic representations of the functions σ_{21} and σ_{23} enable us to write the following inner expansion for $\sigma=r^\nu$ as $\zeta\to\infty$:

$$\sigma=C^\nu x^{\nu m-\delta(\varkappa-1)/\varkappa}\left[c_1\zeta^{(\varkappa-1)/\varkappa}+C_w x^n+\ldots\right],\quad c_1=\frac{\varkappa}{\varkappa+1}p_{21}^{-1/\varkappa} \tag{3.3}$$

Expansion (3.3) enables us to determine the form of the perturbations in the outer solutions. The problem consists of constructing such outer expansions, that the expansion for σ in the region of matching contains a term identical with the second term of the limiting expression (3.3).

Let us specify the line separating the uniform oncoming flow from the outer region, using the expansion

$$r_s=Cx^m(1+C_s x^{m_1}+\ldots) \tag{3.4}$$

where C_s and $m_1<0$ are constants to be defined.

From /2/ it follows that for $m_1>-2\nu/(2+\nu)$ we can assume that (3.4) describes a shock wave and we can require that the Rankine-Hugoniot conditions hold not only for the principal terms of the expansions of the gas-dynamic functions, but also for their perturbations proportional to x^{m_1}. The outer expansions of the flow parameters corresponding to (3.4) have the form

$$\begin{aligned}
v_x&=v_{x0}x^{2(m-1)}[v_{x11}(\eta)+C_s x^{m_1}v_{x12}(\eta)+\ldots]\\
v_r&=v_{r0}x^{m-1}[v_{r11}(\eta)+C_s x^{m_1}v_{r12}(\eta)+\ldots]\\
p&=p_0x^{2(m-1)}[p_{11}(\eta)+C_s x^{m_1}p_{12}(\eta)+\ldots]\\
\rho&=\rho_0[\rho_{11}(\eta)+C_s x^{m_1}\rho_{12}(\eta)+\ldots]\\
w&=w_0x^{2(m-1)}[w_{11}(\eta)+C_s x^{m_1}w_{12}(\eta)+\ldots]\\
r&=Cx^m[r_{11}(\eta)+C_s x^{m_1}r_{12}(\eta)+\ldots]
\end{aligned} \tag{3.5}$$

Substituting these expansions into the Navier-Stokes equations we obtain the following second-approximation system:

$$\begin{aligned}
&\left(\frac{2}{2+\nu}+m_1\right)r_{12}-\frac{2\nu}{2+\nu}\eta\frac{dr_{12}}{d\eta}=\frac{4}{(2+\nu)(\varkappa+1)}v_{r12}\\
&w_{12}=\frac{\varkappa+1}{\varkappa-1}p_{11}\frac{d}{d\eta}(\nu r_{12}r_{11}^{\nu-1})+\frac{p_{12}}{\rho_{11}}\\
&\left(\frac{2+\nu}{2\nu}m_1-\frac{1}{2}\right)v_{r12}-\eta\frac{dv_{r12}}{d\eta}+r_{11}^{\nu-1}\frac{dp_{12}}{d\eta}+(\nu-1)r_{12}\frac{dp_{11}}{d\eta}=0\\
&\frac{p_{12}}{p_{11}}-\varkappa\frac{\rho_{12}}{\rho_{11}}=(2+\nu)(m_1+1)\,\eta^{(2+\nu)m_1/2\nu}\\
&\rho_{11}w_{12}+\rho_{12}w_{11}=p_{12},\quad v_{x12}=\frac{1}{\varkappa+1}(2v_{r12}v_{r11}+\varkappa w_{12})
\end{aligned} \tag{3.6}$$

System (3.6) is of second order (the first two equations can yield another finite relation), and therefore two Cauchy conditions

$$w_{12}(1)=(2+\nu)m_1+2-\nu\frac{dw_{11}}{d\eta}(1),\quad r_{12}(1)=1-\nu\frac{dr_{11}}{d\eta}(1) \tag{3.7}$$

are sufficient to construct its solution. The terms are obtained from the Rankine—Hugoniot relations written for the shock wave (3.4).

Analysis of the asymptotic behaviour as $\eta\to 0$ of the first approximation /4/ and the solutions of system (3.6), enables us to write the limiting form of the outer expansion for $\sigma=r^\nu$ in terms of the variables $x,\ \zeta$

$$\sigma=C^\nu x^{\nu m-\delta(\varkappa-1)/\varkappa}\left[c_1\zeta^{(\varkappa-1)/\varkappa}+C_s c_2 x^{-m_1 n/\nu m}\zeta^{m_1/\nu m+(\varkappa-1)/\varkappa}+C_s c_0 x^{m_1+\delta(\varkappa-1)/\varkappa}+\ldots\right] \tag{3.8}$$

where the constant $c_0(\nu,\varkappa,m_1)$ can be found by numerical integration of system (3.6) with initial conditions (3.7), and

$$c_2=\frac{2\nu(2+\nu)(\varkappa-1)(m_1+1)\,p_{21}^{-1/\varkappa}}{(\varkappa+1)[(2+\nu)\varkappa m_1+2\nu(\varkappa-1)]}$$

The principal terms of the expansions (3.3) and (3.8) are identical /1/, and the matching of subsequent terms requires that one of the correction terms in (3.8) must be identical with the second terms of the inner expansion (3.3). Only the third terms of (3.8) satisfies this requirement, since in this case the powers in ζ match automatically and the choice

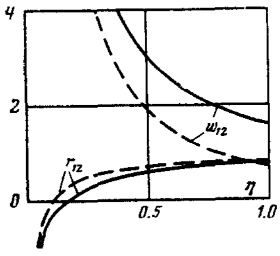

Fig.2

$$m_1 = -\frac{2\nu}{2+\nu} + \frac{\nu}{2(\varkappa+\nu-1)} \tag{3.9}$$

ensures the equality of powers of x. The condition for the coefficients in the expansions (3.3) and (3.8) to be equal, determines the constant

$$C_s = C_w/c_0$$

The last relation implies that the correction terms for the expansions (1.1) depending on the energy E_t and mass M_t discharged by the body in plane-parallel flow, are of the same order x^{m_1} as $x \to \infty$. In axisymmetric flow the analogous corrections caused by E_t are , as before, of order x^{m_1} as $x \to \infty$, but are such greater than the corrections caused by M_t since the coefficient preceding Ψ_0 in expression (3.2) for C_w vanishes when $\nu = 2$.

Fig.2 shows graphs of solutions of system (3.6) with m_1 given by (3.9), satisfying the initial conditions (3.7) for $\varkappa = 1.4$. The solid line corresponds to plane flow ($\nu = 1$) and the dashed line to axisymmetric flow ($\nu = 2$). The values of the constant c_0 are 3.843 and 2.171 for $\nu = 1$ and $\nu = 2$ respectively.

4. Let us now inspect the perturbations of the inner solution (1.2). The description of these perturbations cannot be reduced to the previously introduced correction terms of order x^n. Indeed, comparison of the limit expansions (3.3) and (3.8) makes it essential to introduce into the inner expansions, correction terms of the order of x^{m_2} where

$$m_2 = \left[1 - \frac{2+\nu}{4(\varkappa+\nu-1)}\right]\left[-\frac{2\nu}{2+\nu} + \frac{\nu\varkappa}{2(\varkappa+\nu-1)}\right]$$

Thus we shall seek a solution in the region of the trace, in the form

$$\begin{aligned}
&v_x = C_u x^n v_{x22}(\zeta) + \ldots \\
&v_r = v_{r0} x^{m-1-\delta(\varkappa-1)/\nu\varkappa}\left[v_{r21}(\zeta) + C_s x^{m_2} v_{r22}(\zeta) + \ldots\right] \\
&p = p_0 x^{2(m-1)} p_{21} + \ldots \\
&\rho = \rho_0 x^{-\delta/\varkappa}\left[\rho_{21}(\zeta) + C_s x^{m_2}\rho_{22}(\zeta) + \ldots\right] \\
&w = w_0 x^{2(m-1)+\delta/\varkappa}\left[w_{21}(\zeta) + C_s x^{m_2} w_{22}(\zeta) + \ldots\right] \\
&r = C x^{m-\delta(\varkappa-1)/\nu\varkappa}\left[r_{21}(\zeta) + C_s x^{m_2} r_{22}(\zeta) + \ldots\right]
\end{aligned} \tag{4.1}$$

The auxilliary terms of order x^n introduced earlier, which are of lower order than those introduced later, are omitted from these expansions. The corrections in the expansions for v_x are of no interest, since they are computed only after determining the perturbations in the other parameters of the flow. The correction function in the expansion for the pressure $p_{22}(\zeta) \equiv 0$, since the pressure p is constant in the approximation used, across the trace, and the condition for matching with the outer region demands that $p_{22} \to 0$.

The functions of the second approximation satisfy the linear system of equations .

$$\begin{aligned}
&\left[\frac{2}{2+\nu} - \frac{\varkappa-1}{2(\varkappa+\nu-1)} + m_2\right] r_{22} + n\zeta\frac{dr_{22}}{d\zeta} = \frac{4}{(2+\nu)(\varkappa+1)} v_{r22} \\
&w_{22} = \frac{\varkappa+1}{\varkappa-1} p_{21}\frac{d}{d\zeta}\left(\nu r_{22} r_{21}^{\nu-1}\right) \\
&\frac{K}{nN_{\mathrm{Pr}}}\left\{\frac{d}{d\zeta}\left[r_{21}^{2(\nu-1)}\frac{dw_{22}}{d\zeta}\right] + \frac{d}{d\zeta}\left[2(\nu-1) r_{22} r_{21}\frac{dw_{21}}{d\zeta}\right]\right\} - \\
&\qquad \zeta\frac{dw_{22}}{d\zeta} - \left(\frac{1}{\varkappa} + \frac{m_2}{n}\right) w_{22} = 0, \quad \rho_{21} w_{22} + \rho_{22} w_{21} = 0
\end{aligned} \tag{4.2}$$

the boundary conditions for which are represented by the symmetry conditions for $\zeta = 0$ and the asymptotic matching conditions as $\zeta \to \infty$.

As we know, the symmetry conditions require that there should be no mass, momentum and heat sources on the axis of the trace. In the inner variables these conditions take the following form at $\zeta = 0$:

$$r = v_r = r^{\nu-1}\frac{\partial v_x}{\partial\zeta} = r^{\nu-1}\frac{\partial w}{\partial\zeta} = 0 \tag{4.3}$$

Substituting expansions (4.1) into these formulas, we obtain the condition which must be satisfied at the point $\zeta = 0$ by the functions of the first and second approximation.

In the case of plane symmetry $\nu = 1$ we obtain, for the second-approximation functions,

$$r_{22}(0) = v_{r22}(0) = \frac{dv_{x22}}{d\zeta}(0) = \frac{dw_{22}}{d\zeta}(0) = 0 \tag{4.4}$$

In the case of axisymmetric flow the finiteness of the derivatives $\partial v_x/\partial\zeta$ and $\partial w/\partial\zeta$ represents the sufficient condition for the last two conditions of (4.3) to hold. Therefore, when $\nu = 2$ the last two conditions of (4.4) can be replaced by the condition that the derivatives $dv_{x22}/d\zeta$ and $dw_{22}/d\zeta$ are finite when $\zeta = 0$. The same arguments hold for the first approximation functions.

Using the symmetry conditions obtained we reduce system (4.2) to the linear second-order differential equation for $\sigma_{22} = \nu r_{22} r_{21}^{\nu-1}$,

$$\frac{K}{nN_{\mathrm{Pr}}}\left[\sigma_{21}^{\nu-1}\frac{d^2\sigma_{22}}{d\zeta^2} + (\nu-1)\,\sigma_{22}\frac{d^2\sigma_{21}}{d\zeta^2}\right] - \zeta\frac{d\sigma_{22}}{d\zeta} + N\sigma_{22} = 0 \tag{4.5}$$

$$N = \frac{2+\nu}{4(\varkappa+\nu-1)} - \frac{1}{\varkappa}$$

the solution of which must satisfy the conditions

$$\sigma_{22}(0) = 0;\quad \sigma_{22}(\zeta) = c_2\zeta^N + \ldots,\ \zeta\to\infty \tag{4.6}$$

The functions $v_{r22}, \rho_{22}, w_{22}, r_{22}$ obtained from the solution of problem (4.5), (4.6) by differentiation and using algebraic relations, automatically satisfy all the conditions when $\zeta = 0$ and $\zeta\to\infty$.

We shall now describe a method of constructing the solution of (4.5), (4.6). From the second equation of system (4.2) it follows that the derivative $d\sigma_{22}/d\zeta$ is finite at the point $\zeta = 0$. Analyzing Eq.(4.5) as $\zeta\to\infty$ we find that the following asymptotic representation holds for its solution:

$$\sigma_{22}(\zeta) = a\zeta^N(1 + a_1\zeta^\alpha + a_2\zeta^{2\alpha} + \ldots),\quad \alpha = \frac{(\nu-1)(\varkappa-1)}{\varkappa} - 2 \tag{4.7}$$

where a is an arbitrary constant (the second, linearly independent solution is exponentially small), and the constants $a_1, a_2, \ldots$ can be written explicitly in terms of $\nu, \varkappa, N_{\mathrm{Pr}}$. We first construct the auxilliary solution σ_{22}^* of (4.5) satisfying, at the point $\zeta = 0$, the Cauchy relations $\sigma_{22}^* = 0$, $d\sigma_{22}^*/d\zeta = 1$. Although various reasons prevent numerical integration of this problem up to large values of ζ, nevertheless the use of several terms of the expansion (4.7) yields the constant a^* corresponding to σ^*_{22} with sufficient accuracy. The function $\sigma_{22}(\zeta) = c_2\sigma_{22}^*(\zeta)/a^*$ yields the required solution of the linear equation (4.5) satisfying conditions (4.6).

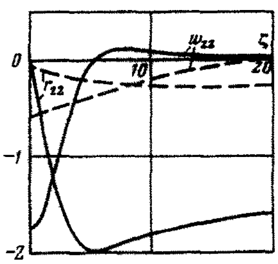

Fig.3

Fig.3 shows graphs of the solutions of system (4.2) for $\varkappa = 1.4$ and $N_{\mathrm{Pr}} = 0.75$ (the solid and dashed lines correspond, respectively, to plane and axisymmetric flow).

In conclusion we note that when $\nu = 1$ a change of variables can be used to reduce Eqs.(4.2) to a degenerate hypergeometric equation /5/ for the function r_{22}. In this case the second-approximation functions are given by the following analytic expressions:

$$v_{x22}(\zeta) = \exp(-\zeta^2/(12K))$$

$$r_{22}(\zeta) = c_2\frac{\Gamma(1-1/(8\varkappa))}{\Gamma(3/2)}\left(\frac{N_{\mathrm{Pr}}}{12K}\right)^{(4\varkappa+1)/8\varkappa}\times\zeta M\left(\frac{4\varkappa+11}{8\varkappa},\ \frac{3}{2},\ -\frac{N_{\mathrm{Pr}}}{12K}\zeta^2\right)$$

$$w_{22}(\zeta) = -c_2\frac{(\varkappa+1)\,p_{21}\,\Gamma(-1/(8\varkappa))}{4\varkappa(\varkappa-1)\,\Gamma(1/2)}\left(\frac{N_{\mathrm{Pr}}}{12K}\right)^{(4\varkappa+1)/8\varkappa}\times M\left(\frac{4\varkappa+1}{8\varkappa},\ \frac{1}{2},\ -\frac{N_{\mathrm{Pr}}}{12K}\zeta^2\right)$$

where $M(a, b, z)$ is the Kummer function. The functions v_{r22} and ρ_{22} can now be found with help of the algebraic relations.

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